$\therefore\ e^{2n-1}v-e^2v_1\left(\dfrac{e^{2n-2}-1}{e^2-1}\right)+ev_1\left(\dfrac{e^{2n-2}-1}{e^2-1}\right)=2v_1.$

$\therefore\ v(e^2-1)e^{2n-1}-ev_1e^{2n-1}+v_1e^{2n-1}=v_1(e^2+e-2).$

$\therefore\ e^{2n-1}=\dfrac{v_1(e+2)}{v(e+1)-v_1}=A$, suppose.

$\therefore\ 2n-1=\log A/\log e$. $\therefore\ n=\frac{1}{2}\log(Ae)/\log e=$the number required.

DIOPHANTINE ANALYSIS.

---

107. **Proposed by L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.**

Required the least three positive integral numbers such that the sum of all three of them, and the sum of every two of them shall be a square number.

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let x, y, and z=the numbers.

Then $x+y=\square=h^2$, $x+z=\square=k^2$, $y+z=\square=l^2$.

$\therefore\ x+y+z=\frac{1}{2}(h^2+k^2+l^2)=\square=s^2$....(1).

$\therefore\ x=s^2-l^2$, $y=s^2-k^2$, and $z=s^2-h^2$.

Put $l=s-m$, $k=s-n$, and $h=s-r$. Substituting these values in (1), we obtain $3s^2-2s(m+n+r)+m^2+n^2+r^2=2s^2$.

Solving for s, we find $s=m+n+r\pm\sqrt{[2(mn+mr+nr)]}$.

Take $mn+mr+nr=mn+r(m+n)=2b^2$.

$\therefore\ s=m+n+x\pm2b$,

$x=2ms-m^2=m(2s-m)$....(2),

$y=2ns-n^2=n(2s-n)$....(3),

$z=2rs-r^2=r(2s-r)$....(4).

Put $n=m+a$. Then $mn+r(m+n)=m(m+a)+r(2m+a)=2b^2$.

$\therefore\ r=\dfrac{2b^2-m(m+a)}{2m+a}$, and $s=\dfrac{(m+a)(3m+a)-am+2b[b\pm(2m+a)]}{2m+a}$.

Substituting in (2), (3), and (4), and multiplying by $(2m+a)^2$, we obtain the following general values:

$$x=m(2m+a)\{2(m+a)(2m+a)-am+4b[b\pm(2m+a)]\},$$
$$y=(m+a)(2m+a)\{(2m+a)^2-am+4b[b\pm(2m+a)]\},$$
$$z=[2b^2-m(m+a)]\{2[b\pm(2m+a)]^2-m(m+a)\},$$
$$x+y=\{(2m+a)[2b\pm(2m+a)]\}^2,$$
$$x+z=\{m^2+2b[b\pm(2m+a)]\}^2,$$
$$y+z=\{(m+a)^2+2b[b\pm(2m+a)]\}^2,$$
$$x+y+z=\{(m+a)(3m+a)-am+2b[b\pm(2m+a)]\}^2.$$

For *positive* values, we have the general condition, $2b^2>m(m+a)$; also, when $b-(2m+a)$ is used, the condition, $b>2m+a$.

When x, y, and z have a common divisor, lowest values are obtained by dividing by the highest common *square* factor.

Multiple values may be obtained by multiplying any set of values of x, y, and z by a *square* number.

m, a, and b may be any integers, subject to the conditions for positive values.

$a=0$, when $m=n$.

The least values are obtained by taking $m=a=1$, and $b=2$, and dividing by 3^2, the highest common square factor.

Whence $x=17$, $y=32$, $z=32$.

These values may also be found by taking $a=0$, and $m=b=1$. Then $x=32$, $y=32$, $z=17$.

The least *different* values are obtained by taking $m=a=1$, and $b=4$, using $b+2m+a$, and dividing by 3^2.

Whence $x=41$, $y=80$, $z=320$.

By using $b-(2m+a)$, in the last case, we find $x=9$, $y=16$, $z=0$.

Excellent solutions of this problem were received from *PROFESSORS ZERR, CROSS*, and *WALKER*, and the late *JOSIAH H. DRUMMOND*. Mr. Cross sent in two solutions, one of which was a solution of the generalized problem. If space permits, his solution of the generalized problem will be published in the next issue of the MONTHLY.

Professor Walker should have been credited with a solution of problem 106.

No solutions of problems 105 and 108 have yet been received.

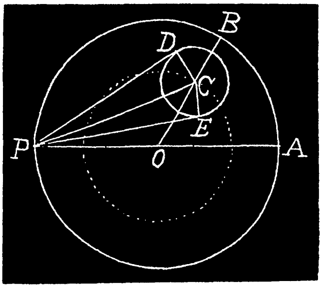
AVERAGE AND PROBABILITY.

90. Proposed by WALTER H. DRANE. Graduate Student, Harvard University.

During a rain-storm a circular pond is formed in a circular field. If a man undertakes to cross the field in the dark, what is the chance that he will walk into the pond?

III. Solution by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

In the following solution, we assume that the path of the man is along a random straight line drawn from a random point in the circumference of the field. We also assume that the number of favorable paths is to the total number of paths as the arc of a circle (radius, the line drawn from the random point on the circumference of the field to the center of the pond) intercepted by the pond, is to the semi-circumference of the same circle, and that all directions of the path are equally probable; that all values of the radius of the pond less than the radius of the field are also equally probable; that all points on the circumference of the field are equally likely to become the point of starting across the field; and that all points of the field are equally likely to become the center of the pond.



Let O be the center of the field, radius $AO=R$; C, the center of the pond; and P, the point where the man enters the field.

Let $x=OC$, the distance from the center of the field to the center of the pond; $z=CD=CE$, the radius of the pond; $\theta=\angle AOB$; and $\phi=\angle CPE=\angle CPD=\sin^{-1}\left(\frac{z}{\sqrt{(R^2+x^2+2Rx\cos\theta)}}\right)$

Then, (1) the chance that the center of the pond lies on the area comprised between two